EFFECT OF FINITE-AMPLITUDE WAVES ON THE EVAPORATION OF A LIQUID FILM FLOWING DOWN A VERTICAL WALL

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Introduction. It is widely known that an evaporating liquid film is a very effective way to cool a hot solid surface because of its small thickness and the closeness of the free-surface temperature to the saturation temperature. For this reason thin films are widely used in technology and the study of the processes and phenomena affecting the temperature gradient across the film is an important problem.

The flow of an isothermal liquid film with a smooth free surface down a vertical plane is unstable at all Reynolds numbers [1]. The growth of long-wavelength perturbations is limited by nonlinear effects and hence finite-amplitude waves are formed. The calculation of such flow regimes using the full set of Navier–Stokes equations is a difficult problem. Two different approaches have been developed using a small parameter inversely proportional to the wavelength, which lead to simplified equations of motion. The first approach [2] leads to a single evolution equation for the perturbation of the free surface. It is derived by expanding all quantities appearing in the original Navier–Stokes equations and the boundary conditions, except for the film thickness, in powers of the small parameter ε. A linear problem is then solved for each order and an evolution equation is obtained after substitution into the kinematic boundary condition. This approach is limited to small-amplitude disturbances and corresponds to small Reynolds number (Re < 1). The second approach [3] averages the equations of motion across the liquid layer. The results [4-6] show good quantitative agreement with the known experimental data over a wide interval of Reynolds number.

The stability of the wave-free flow of evaporating and condensing films down a vertical plane was considered in [7-10]. Evaporation is an additional destabilizing effect which broadens the region where long-wavelength perturbation increase in time. Condensation has the opposite effect and in this case there is a critical Reynolds number below which the wave-free flow smooth free surface of a condensing film is stable. The nonlinear growth of unstable perturbations on evaporating the condensing films was considered in [11] using an approach analogous to [2] for an isothermal film, which is correct for small-amplitude perturbations. The purpose of the present paper is to develop an integrated approach to the study of waves on an evaporating liquid film without making the assumption of small amplitude.

Derivation of the Basic Equations. We consider the flow of a thin layer of viscous liquid down a hot surface in the presence of evaporation from the free surface of the liquid. The flow and the basic notation is shown schematically in Fig. 1. The boundary conditions on the free surface are [11]:

$$
\begin{gathered}
J \equiv \rho\,(\mathbf{v} - \mathbf{v}^{(i)})\,\mathbf{n} = \rho^{(V)}\,(\mathbf{v}^{(V)} - \mathbf{v}^{(i)})\,\mathbf{n},\\
J\,\{r + [(\mathbf{v}^{(V)} - \mathbf{v}^{(i)})\,\mathbf{n}\,]^2/2 - [(\mathbf{v} - \mathbf{v}^{(i)})\,\mathbf{n}\,]^2/2\} + \lambda\nabla T\mathbf{n} - \lambda^{(V)}\nabla T^{(V)}\mathbf{n} +\\
+ (\tau\mathbf{n})\,(\mathbf{v} - \mathbf{v}^{(i)}) - (\tau^{(V)}\mathbf{n})\,(\mathbf{v}^{(V)} - \mathbf{v}^{(i)}) = 0,\\
J\,(\mathbf{v} - \mathbf{v}^{(V)})\,\mathbf{n} - (\tau - \tau^{(V)})\,\mathbf{nn} + p - p^{(V)} = 2H\sigma,\\
J\,(\mathbf{v} - \mathbf{v}^{(V)})\,\mathbf{t} - (\tau - \tau^{(V)})\,\mathbf{nt} = 0, \quad (\mathbf{v} - \mathbf{v}^{(V)})\,\mathbf{t} = 0,\\
\mathbf{n} \equiv (-\partial h/\partial x,\ 1)/N, \quad \mathbf{t} \equiv (1,\ \partial h/\partial x)/N, \quad 2H \equiv -\partial^2 h/\partial x^2/N^3,\\
N = \sqrt{1 + (\partial h/\partial x)^2}.
\end{gathered}
$$

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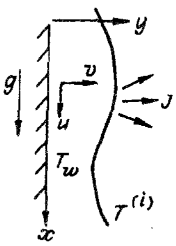

Fig. 1

Here J is the mass flux as a result of evaporation from a unit area of the surface per unit time, h is the instantaneous thickness of the film measured from the solid surface, σ is the surface tension, r is the heat of vaporization, $\mathbf{v}^{(i)}$ is the velocity of the particles on the free surface; quantities with superscripts correspond to the vapor and quantities without superscripts correspond to the liquid; $\mathbf{v} \equiv (u, v)$ is the velocity vector, T is the temperature, p is the pressure, τ is the viscous stress tensor, ρ is the density, λ is the thermal diffusivity. The first equation represents the definition of the mass flux J and its conservation law, the second, third, and fourth equations are the conservation laws for the energy and the two components of the momentum, respectively.

Here and below the temperature dependence of the physical parameters of the liquid will be neglected (we consider small temperature differences). Taking the limits $\rho^{(v)}/\rho \to 0$, $\mu^{(v)}/\mu \to 0$ and $\lambda^{(v)}/\lambda \to 0$ (μ is the dynamical viscosity), the equations of motion and the boundary conditions can be written in dimensionless form as:

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{\varepsilon}{\mathrm{Re}} \frac{\partial^2 u^*}{\partial x^{*2}} + \frac{1}{\varepsilon\,\mathrm{Re}} \left(\frac{\partial^2 u^*}{\partial y^{*2}} + 3 \right),$$

$$\varepsilon^2 \left(\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} \right) = -\frac{\partial p^*}{\partial y^*} + \frac{\varepsilon}{\mathrm{Re}} \left(\varepsilon^2 \frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right),$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \quad \frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\varepsilon\,\mathrm{Re}\,\mathrm{Pr}} \left(\frac{\partial^2 T^*}{\partial y^{*2}} + \varepsilon^2 \frac{\partial^2 T^*}{\partial x^{*2}} \right),$$

$$u^* = v^* = 0, \quad T^* = 1, \quad y^* = 0,$$

$$\frac{J^*}{\varepsilon\,\mathrm{Ku}_m} = v^* - \frac{\partial h^*}{\partial t^*} - u^* \frac{\partial h^*}{\partial x^*}, \quad y^* = h^*(x^*, t^*),$$

$$J^* + \frac{J^{*3}}{(\mathrm{Ku}_m D)^2 \varkappa} - \frac{\varepsilon^2}{N} \frac{\partial h^*}{\partial x^*} \frac{\partial T^*}{\partial x^*} + \frac{1}{N} \frac{\partial T^*}{\partial y^*} +$$

$$+ \frac{4\varepsilon J^*}{\varkappa\,\mathrm{Re}\,N^2} \left[\frac{\partial u^*}{\partial x^*} \left(\varepsilon^2 \left(\frac{\partial h^*}{\partial x^*} \right)^2 - 1 \right) - \frac{\partial h^*}{\partial x^*} \left(\frac{\partial u^*}{\partial y^*} + \varepsilon^2 \frac{\partial v^*}{\partial x^*} \right) \right] = 0,$$

$$-\frac{J^{*2}}{(\mathrm{Ku}_m)^2 D} - \frac{2\varepsilon}{\mathrm{Re}\,N^2} \left[\frac{\partial u^*}{\partial x^*} \left(\varepsilon^2 \left(\frac{\partial h^*}{\partial x^*} \right)^2 - 1 \right) - \frac{\partial h^*}{\partial x^*} \left(\frac{\partial u^*}{\partial y^*} + \varepsilon^2 \frac{\partial v^*}{\partial x^*} \right) \right] + p^* =$$

$$= -\frac{3^{1/3} \varepsilon^2 \mathrm{Fi}^{1/3}}{\mathrm{Re}^{5/3} N^3} \frac{\partial^2 h^*}{\partial x^{*2}}, \quad -4\varepsilon^2 \frac{\partial h^*}{\partial x^*} \frac{\partial u^*}{\partial x^*} + \left(\frac{\partial u^*}{\partial y^*} + \varepsilon^2 \frac{\partial v^*}{\partial x^*} \right) \left(1 - \varepsilon^2 \left(\frac{\partial h^*}{\partial x^*} \right)^2 \right) = 0,$$

$$\mathrm{Ku}_m \equiv \mathrm{Re}\,\mathrm{Pr}\,\mathrm{Ku}, \quad N = (1 + \varepsilon^2 (\partial h^*/\partial x^*)^2)^{1/2},$$

where $u^* = u/u_0$; $v^* = v/(\varepsilon u_0)$; $y^* = y/h_0$; $x^* = \varepsilon x/h_0$; $t^* = \varepsilon u_0 t/h_0$; $J^* = h_0 rJ/\lambda \Delta T$; $\Delta T = T_w - T_s$; $T^* = (T - T_s)/\Delta T$; $p^* = p/\rho u_0^2$; $\mathrm{Re} = u_0 h_0/\nu$; $\mathrm{Pr} = \nu/k$; $\mathrm{Ku} = r/C_p \Delta T$; $\varkappa = 2r/u_0^2$; $\mathrm{Fi} = (\sigma/\rho)^3/g\nu^4$; $D = \rho^{(v)}/\rho$; $u_0 = gh_0^2/3\nu$; $\varepsilon = h_0/L$; L is the scale of the wave motion along the x coordinate, T_w is the wall temperature, T_s is the vapor saturation temperature, and k is the thermal conductivity.

We consider long-wavelength perturbations ($\varepsilon << 1$). Since $\varkappa$ is usually very large, the original equations can be simplified considerably. After integration perpendicular to the layer they take the form

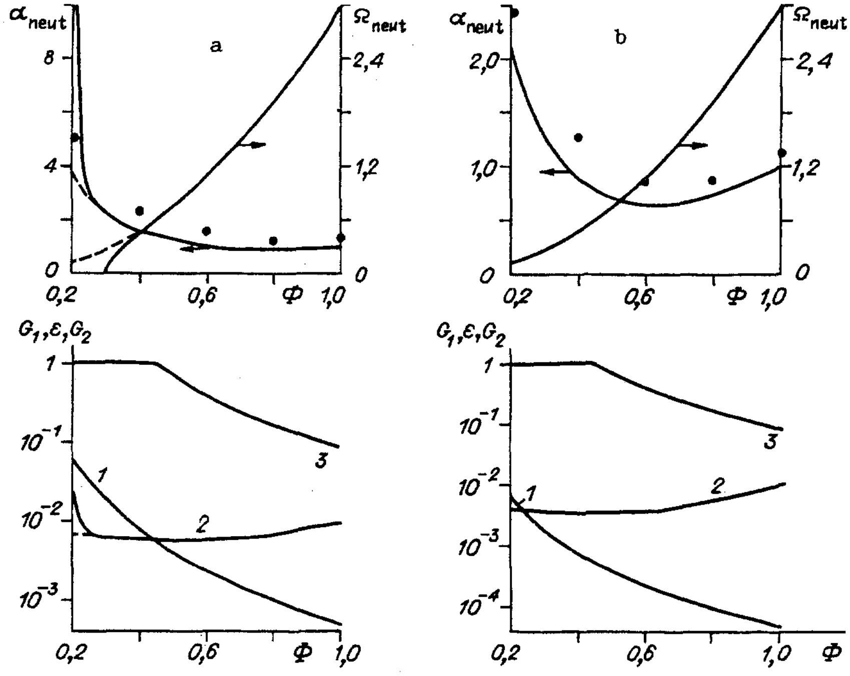

Fig. 2

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\int_0^h u^2 dy = \frac{3^{1/3}\mathrm{Fi}^{1/3}}{\mathrm{Re}^{5/3}} h\frac{\partial^3 h}{\partial x^3} - \frac{h}{\mathrm{Ku}_m^2 D}\frac{\partial J^2}{\partial x} - \frac{J}{\mathrm{Ku}_m} u\big|_{y=h} + \frac{1}{\mathrm{Re}}\left(3h - \frac{\partial u}{\partial y}\bigg|_{y=0}\right), \tag{1}$$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = -\frac{J}{\mathrm{Ku}_m}, \quad q = \int_0^h u dy, \quad \frac{\partial^2 T}{\partial y^2} = 0, \quad J = -\frac{\partial T}{\partial y}\bigg|_{y=h}, \quad T\big|_{y=0} = 1.$$

Here the asterisks on the dimensionless quantities have been omitted and the quantity ε has been set equal to unity. We note that in spite of the fact that we have discarded the convective terms in the heat equation, the inclusion of these terms in the first equation of (1) is correct, since the term in the parentheses in this equation is small. We also note that the term involving the capillary pressure should be kept if $\mathrm{Fi} \sim \mathrm{Re}^5/\varepsilon^6$, which is correct for a large number of liquids. Hence it follows that (1) is correct for the evolution of perturbations if $\varepsilon << \min(1, \mathrm{Re\,Pr})$ and $\mathrm{Fi} \sim \mathrm{Re}^5/\varepsilon^6$.

We assume the velocity profile $u(x, y, t) = (3q(x, t)/h(x, t))\,(y/h(x, t) - y^2/2h^2(x, t))$ and the condition $T|_{y=h} = 0$ on the free surface. Wave-free flow corresponds to the solution $q = \Phi^3$, $h = \Phi = (1 - 4x/3\ \mathrm{Ku}_m^0)^{1/4}$, where Ku_m^0 is calculated in the initial cross section of the flow. Hence we have an unperturbed solution that is inhomogeneous in space. The problem reduces to the linear stability analysis of this solution and to the study of the possible nonlinear evolution of unstable perturbations. Making the substitutions $q \to \Phi^3 + q$, $h \to \Phi + h$ in (1), we obtain a system of nonlinear partial differential equations whose coefficients depend on x. Rigorous mathematical approaches, and hence numerical algorithms, are not available for perturbations evolving in both space and time. Therefore we consider the case of small temperature differences ($\mathrm{Ku}_m^0 >> 1$) and the linear stability and different nonlinear solutions which are periodic in x, over the background of a slowly-varying unperturbed flow. Hence we introduce two different scales of length in the x direction. The first scale is determined by the variation in Φ. This approach (the quasi-parallel approximation) is often used in problems with inhomogeneities in the spatial variable and is correct when $|\partial h/\partial x| \sim \varepsilon >> |d\Phi/dx| = 1/(3\Phi^3\,\mathrm{Ku}_m^0)$, i.e. the wavelength λ of the periodic perturbations must be much smaller than the typical distance over which the unperturbed solution changes significantly.

With the help of the transformation $x \to (\mathrm{Fi}/9\ \mathrm{Re}^5)^{1/6}x$, $t \to (\mathrm{Fi}/9\ \mathrm{Re}^5)^{1/6}t$, we obtain the following basic system of equations

$$\frac{\partial q}{\partial t} + 1{,}2\frac{\partial}{\partial x}\frac{(q+\Phi^3)^2}{h+\Phi} + \frac{1{,}2}{K}\left(\frac{5\Phi}{3} + \frac{(q+\Phi^3)^2}{3\Phi^3(h+\Phi)^2} - \frac{2(q+\Phi^3)}{\Phi(\Phi+h)}\right) =$$

$$= Zh - (Z + 1{,}5/K)\left(\frac{q+\Phi^3}{(h+\Phi)^2} - \Phi\right) + 3\left(\Phi\frac{\partial^3 h}{\partial x^3} + h\frac{\partial^3 h}{\partial x^3} - \frac{21}{(3K)^3}\frac{h}{\Phi^{11}}\right) +$$

$$+ \frac{2}{D(\mathrm{Ku}_m^0)^2}\left(\frac{1}{3K\Phi^5} + \frac{1}{(\Phi+h)^2}\left(\frac{\partial h}{\partial x} - \frac{1}{3K\Phi^3}\right)\right), \quad \frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \frac{1}{K}\left(\frac{1}{\Phi} - \frac{1}{\Phi+h}\right), \quad \langle q\rangle = 0. \tag{2}$$

Here the thickness h and the flow rate q are assumed to be periodic in x; $Z = (81\ \mathrm{Fi}/\mathrm{Re}^{11})^{1/6}$; $K = (9\ \mathrm{Re}^5/\mathrm{Fi})^{1/6}\mathrm{Ku}_m{}^0$; the linear scale of all quantities and Re is defined by the initial cross section of the flow; Φ characterizes the distance from the initial cross section (the large-scale spatial variable) and varies from 1 to 0 for evaporation ($K > 0$) and from 1 to ∞ for condensation; $\langle q\rangle = (1/\lambda)\int_0^\lambda q(x, t)dx$ is the flow rate averaged over the wavelength. To better understand the physical meaning of the parameters Z and K, we note that they are simply related to easily measurable quantities: $\mathrm{Re}/\mathrm{Ka} = 81^{1/11}/Z^{6/11}$ and $\mathrm{Pr\ Ku} = KZ/3$ ($\mathrm{Ka} = \mathrm{Fi}^{1/11}$ is the Kapitza number).

Wave-free flow corresponds to the solutions $q = 0$, $h = 0$ of (2). To study its stability against spatially periodic perturbations $\sim \exp[i\alpha(x - \Omega t)]$ we linearize the system of equations (2). Then from the condition for the existence of a solution it is not difficult to show that perturbations with $\alpha < \alpha_{\mathrm{neut}}$ grow in time, while those with $\alpha > \alpha_{\mathrm{neut}}$ damp out. The expressions for α_{neut} and the velocity Ω_{neut} of neutral perturbations are quite complicated

$$\Omega_{\mathrm{neut}} = \frac{A_1C_1 - B_2}{A_2 + C_1},$$

$$\alpha^2_{\mathrm{neut}} = \frac{\Omega^2_{\mathrm{neut}} - A_1\Omega_{\mathrm{neut}} - B_1 \pm [(\Omega^2_{\mathrm{neut}} - A_1\Omega_{\mathrm{neut}} - B_1)^2 + 4D_1A_2C_1]^{0,5}}{-2D_1},$$

$$A_1 = 2{,}4\Phi^2, \quad C_1 = -1/K\Phi^2, \quad A_2 = -0{,}1/K\Phi^2 + Z/\Phi^2, \quad D_1 = -3\Phi, \tag{3}$$

$$B_1 = -1{,}2\Phi^4 - 2/(D\Phi^2(\mathrm{Ku}_m^0)^2),$$

$$B_2 = -3Z - 1{,}4/K + 7/(3K^3\Phi^{11}) - 4/(3DK\Phi^6(\mathrm{Ku}_m^0)^2)$$

and are difficult to use analytically. Since $\mathrm{Ku}_m{}^0 >> 1$ for the plane-parallel approximation to be applicable, (3) can be simplified

$$\Omega_{\mathrm{neut}} \approx 3\Phi^2, \quad \alpha^2_{\mathrm{neut}} \approx \Phi^3/2 \pm [\Phi^6/4 + Z/(3K\Phi^5)]^{0,5}. \tag{4}$$

For an evaporating film ($K > 0$) only one of the roots α_{neut} is chosen in (3) and (4), while for condensation ($K < 0$) there exists a critical value of Z for the onset of wave formation and both branches of α_{neut} are physically meaningful.

To estimate the validity of the assumptions made in deriving (2), we take the wavelength of neutral perturbations as the longitudinal scale L and obtain

$$G_1 = 1/(3\Phi^3|\mathrm{Ku}_m^0|) \ll \varepsilon = (9\mathrm{Re}^5/\mathrm{Fi})^{1/6}\Phi\alpha_{\mathrm{neut}}/2\pi \ll G_2 = \min(1, 1/(\Phi^3\mathrm{Pr\,Re})).$$

The neutral stability curves, phase velocities, and the quantities G_1, ε, and G_2 are shown in Fig. 2 (curves 1-3) as functions of the parameter Φ for PrKu = 100 and 1000 (a and b, respectively), calculated from (3) (solid curves) and from the approximate formula (4) dashed curves). The physical characteristics ($\mathrm{Pr} = 1.73$, $\mathrm{Ka} = 13.75$, and $D = 6.24\cdot 10^{-4}$) correspond to water at 100°C and Re/Ka = 0.5. As clearly shown by the graphs, (4) is quite accurate and differences show up only for comparatively small Φ, where the plane-parallel approximation becomes inapplicable ($\varepsilon < G_1$).

It was verified that the calculated results are within the limits of applicability of the approximations used.

The points in Fig. 2 correspond to the results of [9], where the neutral wave-number lines were calculated using the Orr–Sommerfeld equation for the problem. The agreement is satisfactory, in our opinion and supports the correctness of the integrated approach.

Numerical Calculation of the Nonlinear Regimes. We consider solutions of (2) of the form $q = q(\xi)$, $h = h(\xi)$, $\xi = x - ct$ (c is the phase velocity) which are periodic in the coordinate ξ with period $\lambda = 2\pi/\alpha$ (α is the wave number). Using the finite Fourier series representation

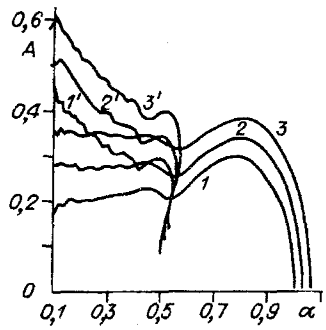

Fig. 3

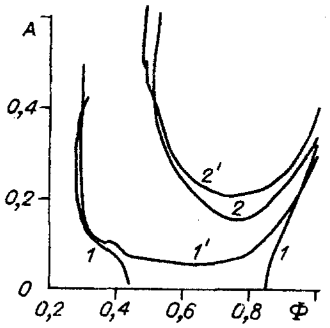

Fig. 4

$$(q, h) = \sum_{n=-N}^{N} (q_n, h_n) \exp [i\alpha n \xi], \quad (5)$$

after substitution into (2) we obtain a system of nonlinear algebraic equations. The Fourier harmonics of the nonlinear terms in (2) were calculated using the pseudospectral method. The phase velocity was determined by using the invariance of (2) to shifts in the origin of the coordinate system, and hence the phase of one of the harmonics in (5) can be assumed as given. The Newton–Kantorovich iteration method was used to solve the nonlinear system of algebraic equations for different values of the parameters Z, K, and α.

The results of [5, 6] for flow of an isothermal film were used to test the algorithm and also as initial data for large values of K. Using a small stepsize in the parameters, the solution was calculated over a wide region of the parameter space. The calculation was tested by verifying that the last harmonic of (5) satisfies the condition

$$|h_N| / \sup_{-N<n<N} |h_n| < 10^{-3}.$$

Calculated Results for Nonlinear Regimes. There are three external parameters in the problem: Re/Ka, PrKu, and Φ. For fixed values of these parameters the calculations show that there exist different one-parameter families of nonlinear steady-state travelling-wave solutions. Inside each of the families, near the singular points, the parameter can be taken as the wave amplitude, while far from the bifurcation points the wave number α ($\alpha = 2\pi/\lambda$, where λ is the wavelength of the nonlinear solution) can be used as the parameter.

We consider only two of the large number of possible families of solutions. The first family of waves appears when the trivial solution h = q = 0 becomes unstable. For Φ not too close to unity this wave solution exists in the region from $\alpha = \alpha_{neut}$ to very small α, as considered in the present paper. In Fig. 3 curves 1-3 show the amplitudes of the solutions of the first family as functions of the wave number for PrKu = 1000, 100, and 50, respectively. Here Re/Ka = 0.5 and $\Phi = 1$, and the amplitude is defined as the difference between the maximum and minimum thicknesses ($A = |h_{max} - h_{min}|$). The amplitudes for waves of the second family are shown by curves 1′-3′. Waves of the second family appear when waves of the first family with double the spatial period become unstable. They have also calculated up to very small values of α for $\Phi \leq 1$.

Other families of solutions were not considered because it is not possible to discuss all solutions and also because the results for an isothermal film (PrKu = ∞) [5, 6] show that except for wave solutions of the first two types, all of these solutions are unstable over the entire region of wave numbers.

It follows from Fig. 3 that the amplitudes of waves from the same period increases with increasing temperature difference and also that the function $A(\alpha)$ is qualitatively similar for the same family for different values of PrKu.

Figure 4 shows the wave amplitude as a function of the parameter Φ, which models the growth of the wave structure in the flow along an evaporating film. Here Re/Ka = 0.5 and curves 1 and 2 correspond to waves of the first family with $\alpha = 0.8$ and PrKu = 1000 and 100, respectively. Curves 1′ and 2′ correspond to waves of the second family with $\alpha = 0.3$ and with the same values of PrKu and Re/Ka. The two unconnected branches of curves 1 result from the behavior of $\alpha_{neut}(\Phi)$ in Fig. 2: when $0.44 < \Phi < 0.84$ the neutral wave number $\alpha_{neut} < 0.8$ and perturbations with $\alpha = 0.8$ lie in the linear stability region.

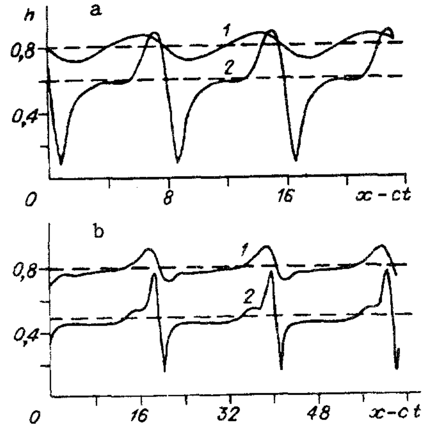

Fig. 5

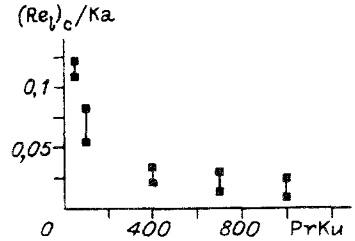

Fig. 6

The build-up of finite-amplitude waves in an evaporating film is determined by two basic factors: 1) the decrease in the intensity of the wave process with decreasing Reynolds number, as in the case of an isothermal film, and 2) the opposite tendency due to the presence of a phase transition on the interface. The competition between these two effects explain the extrema observed in Fig. 4. Beginning with a certain Φ the wave amplitude increases rapidly and for $\Phi < \Phi_c$ (α, PrKu) the nonlinear wave regime does not exist. To see more clearly what happens when $\Phi = \Phi_c$, Fig. 5a, b shows typical thickness profiles corresponding to the parameters of curves 2 and 2′ of Fig. 4. Here the dashed lines correspond to the value of Φ for which the solution is given. In Fig. 5a, b the profile 1 is for $\Phi = 0.8$, while profile 2 is for a small neighborhood about $\Phi = \Phi_c$. Upon further movement along curves 2 and 2′ of Fig. 4 there is a rapid growth in the maximum film thickness approaches zero, corresponding to the formation of a "dry" spot.

We conclude from the above results that the most important effect caused by waves in the formation of dry spots with decreasing Φ. A detailed study of this effect is important in applications because it limits the maximum possible heat flux that can be carried off from a hot surface.

Figure 6 shows the calculated quantity Φ_c^3 Re/Ka $\equiv$ $(\mathrm{Re}_l)_c$/Ka (Re_l is the "local" wave-averaged Reynolds number) for different values of PrKu. It is not difficult to obtain from these results the minimum flow rate and film thickness for which the wave structure is possible for a given liquid.

When $\Phi < \Phi_c$, dry spots form, as shown above. Because of the internal parameter α and the existence of different families of waves, an interval of Φ_c exists at fixed PrKu. The calculations show that the lower boundary of Φ_c is usually determined by long waves belonging to the second family, while the upper boundary is determined by short waves of the first family.

It would be interesting to test the theoretical results by comparing them with experiments on the formation of dry spots in evaporating liquid films. Because of the smallness of the Reynolds number this problem is quite difficult to study experimentally. We know of only one experimental paper [12] devoted to this problem. Unfortunately, the measurements were done only on insufficiently heated liquids.

Conclusions. The growth of unstable long-wavelength perturbations on the surface of an evaporating liquid film can lead to different steady-state travelling-wave flow regimes. The wave flow regime exists only up to certain critical values of the film thickness, and furthermore there is the apparent formation of dry spots. As the film thickness changes during the evaporation process from the initial value to the critical value the amplitude of steady-state waves passes through a minimum and increases rapidly near the critical film thickness. The critical film thickness depends on the temperature drop across the films and also the wavelength and type of wave regime. By controlling these parameters the instant of film rupture can be delayed.

The approach developed here for waves on the surface of an evaporating film can easily be extended to nonequilibrium evaporation from a free surface and the Marangoni effect, which is important for thin films when the temperature of the free surface can vary.

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